A Sharing Platform for Multi-Tenant PONs

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***Abstract*—In this paper we address the sharing incentive issue in multi-tenant passive optical networks (PONs). We propose an economic-robust and efficient sharing platform for new emerging multi-tenant PON networks. This platform is capable of accommodating a diverse range of service providers and enhancing the network utilization. We propose a sharing platform that provides sharing incentives for the incumbent network operators through monetization of inter-operator network sharing. Meanwhile, the platform allows the incumbent operators to operate a virtual instance of the bandwidth scheduling algorithm which enables them to meet their quality of service and latency requirements. Therefore, the proposed sharing platform grants a high degree of control to the operators co-operating the same network while, thanks to the higher resource efficiency, reduces the initial investment. We first model the multi-tenant PON as a market and define the roles of the virtual network operators (VNOs) and the infrastructure provider (InP) along with their utility functions. We propose a double auction mechanism to facilitate the trading of excess resources. The proposed double auction satisfies the crucial economic properties of a market while it achieves more efficient resource allocation among the market players. We have theoretically proven the economic robustness of the mechanism including incentive compatibility, individual rationality and weak budget balance. Through extensive market simulations, we confirmed that the proposed mechanism achieves superior allocative efficiency compared to a reference baseline mechanism.**

***Index Terms*—PON, Auction, Network Sharing, Infrastructure Sharing.**

I. Introduction

Thanks to the advances in optical networks technologies, the standardization bodies are moving toward higher rates for passive optical networks (PON). The IEEE 100G-EPON Task Force [1] is aiming at 100 Gbit/s of capacity for its next-generation of PON and the International Telecommunication Union's (ITU) NG-PON2 [2] is already offering 80 Gbit/s capacity using Time and Wavelength Division Multiplexing (TWDM).

Currently, PONs are mainly dedicated to fiber to the home (FTTH) systems, e.g., for providing residential broadband. However, PONs have attracted interest from a diverse range of service providers (e.g., entities offering fifth generation (5G) mobile communications serivces, and Virtual Reality (VR) services [3]) as they are interested in using the PON as their access platform. The advantages that PON offers compared to the other access solutions include high-capacity transmitting, lower CAPEX and OPEX costs compared to other alternatives such as point-to-point fiber as PON enables multiplexing gains from sharing the fiber capacity between the optical network units (ONUs).

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The network operators are seeking cost-effective ways to accommodate new services and meet the users' accelerating demand for network capacity. One key challenge remains to be the traditional sole ownership of the network that becomes highly cost-inefficient as more expensive equipment and transmission media (e.g., optical fibre) has to be deployed. Thus, new joint ownership models are becoming appealing to the operators as they can considerably lower total cost of network ownership.

Sharing the network reduces redundant CAPEX by splitting the infrastructure investment as well as the OPEX through economy of scale for the operators, leading to lower *"cost per bit"*.

In the context of optical access networks, such cost reduction can be achieved by increasing infrastructure utilization. One approach to sharing the infrastructure is the passive approach in which the operators share the site and the passive equipment. The second approach, which will be our primary focus in this research, is active infrastructure sharing, which enables improvements in network utilization by more fine-grained sharing models. For instance, sharing a PON by dedicating a full wavelength channel to each different virtual network operator is possible using the currently available technologies. However, such coarse-grained sharing models impose boundaries on the extent of sharing, as it does not allow to utilize any unused capacity within each wavelength channel. Sharing the network at the sub-wavelength scale is thus important to fully exploit the advantages of multi-tenancy.

Traditional sharing models include sharing of capacity at higher layers, where for instance, a VNO can collect traffic from its customers at a metro or regional aggregation point. This is the type of multi-tenancy service offered for example through bitstream. The main issue is its lack in flexibility as the VNO can only offer very simple type of services to its customers, with little ability to differentiate its offer from its competitors, e.g., in terms of capacity, latency or quality of service. In short, current sharing models fail to provide the operators sufficient control over the shared infrastructure.

The main contributions of this paper are as follow:

1) We propose the use of auction algorithms for resource allocation of multi-tenant optical networks. To the best of our knowledge, this is the first time that this approached is used.
2) We propose a multi-item double auction mechanism which while maintaining the typical desired economic properties for a market place (details are provided in section 2), achieves a higher allocative efficiency when compared with state of the art mechanisms used in wireless spectrum sharing [4].

The rest of the paper is organized as follows: First, we give a brief overview of the PON Dynamic Bandwidth Allocation

(DBA) mechanism in section II. We model the multi-tenant PON network as a market in section III, while introducing the roles of the agents involved in the market. We will then describe the desirable economic properties of the market. Section IV provides a glance into the State-of-the-art on double auction mechanisms. Then we will present our proposed double auction mechanism for the multi-tenant PON market in section V. The theoretical proofs for the satisfaction of the economic properties are presented in section VI. Then we will present the analysis of the allocative efficiency of the proposed mechanism in section VII. Finally, we will conclude the work in section VIII.

II. Dynamic Bandwidth Allocation (DBA)

PON is a point-to-multipoint optical access technology, which requires scheduling in the upstream transmission to avoid collisions between the data sent by the ONUs. DBA (Dynamic Bandwidth Allocation) is a process that assigns time slots to each ONU for upstream transmission. The outcome of the DBA process is the transmission schedule for the ONUs, i.e., "Bandwidth Map (BWmap)". Fig. 1 depicts the format of the XGS-PON [5] BWmap partition and an allocation structure. The BWmap is generated by the OLT and broadcast to all of the ONUs in every 125 microseconds (i.e., the duration of a PON frame). The finest granularity allowed for each allocation structure is 16 bytes. Thus, each BWmap may contain up to 9720 allocation structures for a 10 Gb/s upstream channel. ITU standards identify two DBA classifications. The first type is status reporting (SR) DBA which schedules the transmission based on the definite reports of buffer occupancy of the ONUs and generates a precise allocation based on it. The second type is non status reporting (NSR) DBA which bases the scheduling on the information acquired from traffic monitoring. The SR DBA provides higher precision but imposes some latency due to the exchange of control signals. Current PONs fail

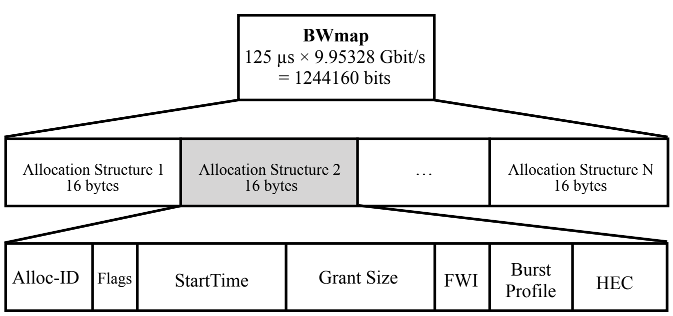


Fig. 1: XGS-PON BWmap, allocation structure format

to support the diverse range of requirements associated with next generation of services (for 5G and beyond). For instance, current PONs cannot be used to provide connectivity between remote radio-head (RRH) and base band unit (BBU) in mobile Cloud-RAN applications, as it cannot meet the required delay budget, which is of the order of few hundreds microseconds [6]. Thus, the research community is investigating new DBA algorithm designs e.g. predictive [3] or unified PON-Wireless scheduler [6], to provide support for these ultra low-latency services.

The research community is currently investigating the possibility to virtualize the DBA [7], [8] to increase its level of programmability. When paired with a larger scale virtualization framework, such as for example the cloud central office (cloud-CO) [9] or the central office rearchitected as a data centre [10], this can facilitate the co-existence of diverse service providers on the same platform and allow them to run their own flavor of the DBA to support specific services (including low-latency, etc).

In [11] the authors have proposed an architecture to implement flexible and cost-efficient Virtual PON systems. Their proposed *software-OLT* architecture enables rapid deployment of the OLT functions and reduces the OLT management costs and power consumption as it consolidates the majority of the OLT functions in commodity servers. They also address technical OLT virtualization issues, including the upstream frame loss caused by the processing time jitter by proposing a collision detection and variable guard interval to solve it.

In [3] the authors have proposed a new Cloud-based PON access architecture. They have proposed the separation of the control plane and data plane functions of the OLT and the consolidation of the control plane function in a datacenter. Then they have tackled the delay increase problem by introducing a predictive DBA pre-allocation algorithm which reduces the disruptive effects of the imposed delay. However, enabling multi-tenancy has not been the main focus in any of the above-mentioned papers.

In our prior work [7] we have introduced a concept where multiple virtual network operators (VNOs), each providing a different service, can coexist on the same PON network while running a virtual instance of the DBA algorithm of their choice and aggregating these scheduling decisions into a final bandwidth map. Our results showed that DBA virtualization can considerably improves the PON utilization by enabling capacity sharing across VNOS within each upstream frame. Nevertheless, this work overlooks the incentives of the VNOs to share their excess bandwidth with other VNOs. In the next section, we propose a solution to this issue by introducing a network model for the inter-operator bandwidth allocation in PONs and a sharing incentive mechanism based on auctions.

III. The Market Model

In situations where the participants lack incentives for sharing a resource, *monetization* (i.e., monetary compensation) comes to the help as a manifest tool to incentivize self-interested agents to engage in sharing. The same tool can be used to address the incentive problem in multi-tenant PONs, where VNOs can be compensated by money in return for sharing their excess capacity with others. The VNOs in demand of extra capacity will report their valuations while on the other side the VNOs who own this extra capacity will announce their valuation and then simply, the demander with the highest valuation gets the capacity from the supplier with the lowest. However, such simple solution could lead to a sub-optimal market, as it can allow the VNOs to strategize and take advantage of auction design flaws to improve their own payoff by damaging the others. In this work, we will study

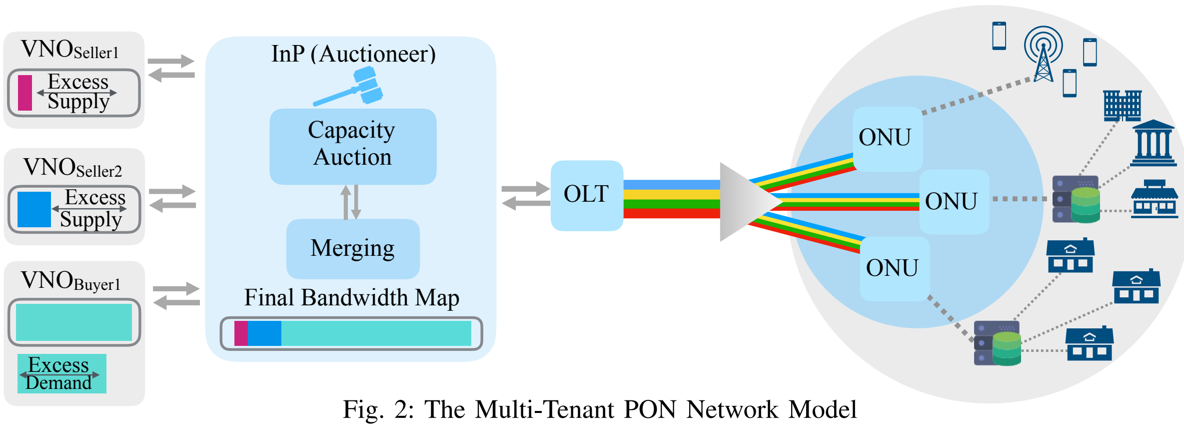


Fig. 2: The Multi-Tenant PON Network Model

the potential design flaws and will try to build a mechanism to minimize or entirely eliminate them.

In this section, we present a market model for the multi-tenant PON. We first introduce the market players and the preliminaries while defining the essential parameters and features of the market. Next, we point out the desired economic properties of the market and define the utility function of the market players.

A. Preliminaries

The market consists of a set of M sellers $\mathbb{S} = \{s_1, s_2, ..., s_i\}$, $i \in M$ and a set of N buyers $\mathbb{B} = \{b_1, b_2, ..., b_j\}$, $j \in N$ and one auctioneer. This constructs a two-sided market in which a number of traders are competing to trade identical items in a way that it will maximize their payoff. There is also an auction maker (broker) present that is responsible for operating the auction. In our market, the InP plays the role of the auctioneer.

The auctioneer initiates the auction. Each seller announces the quantity of the items offered q_i^S along with the per-item (the finest granularity on PON's upstream allocation structure) ask value v_i^S to the auctioneer. Simultaneously the buyers will send their pair of the number of items required q_j^B and the bid value v_j^B to the auctioneer. The ask and bid values (v_i^S, v_j^B) are $\in$ (0, 1). The auction mechanism is common knowledge, i.e., is known to all the VNOs and the InP.

Definition 1. *(VNOs' Per-Item Valuation). The valuation of a VNO for an item (a 16 bytes frame block) is driven by the probability (p) of that VNO being able to utilize the item. This valuation thus ranges between [0,1] , $p = 0$ for definitely not having any ONU asking for any bandwidth, $p \in (0, 1)$ for having a probability between 0 and 1 that a new upstream burst arrives in one of the users' buffer since the last time that the buffer occupancy was reported (this might be predicted using machine learning or traffic history monitoring techniques), and $p = 1$ for definite demand from the users that in PON is referred to as "buffer occupancy reports". For example, if a VNO reports a quantity of 1000 items and its preassigned provisioned share is 900 items, it means that it will certainly utilize the 900 items with the probability of 1 and is demanding for 100 extra items and there is a probability of v_j that it will utilize this item. However, our focus is not the valuation process of the VNOs, i.e., the VNOs can use more sophisticated methods to value an item.*

Each seller S_i is ready to supply q_i^S items for the minimum price of v_i^S. On the Other side of the market each buyer $B_j (j \in J)$ is willing to buy q_j^B items and is willing to pay q_j^B for each item at most.

After the auction is finished the winner sellers will each receive $p^{\mathrm{S}} \times \theta_i^{\mathrm{S}}$ and the winner buyers will pay $p^{\mathrm{B}} \times \theta_j^{\mathrm{B}}$ with the p^{S} and p^{B} as the buyers' and the sellers' trade price and θ_i^{S} and θ_j^{B} as the quantity of the items traded for the buyers and the sellers respectively. By our design the traders can be partially satisfied that is they can win/sell $\theta_i^S \leq q_i^S$ or $\theta_j^B \leq q_j^B$.

Definition 2. *(Strategy). In the context of our market, the only way that a trader can change the outcome of the market and the final allocation is through its reported value and quantity. Thus a trader's strategy is the value and quantity pair that it reports to the auctioneer. A strategy is a "dominant strategy" if regardless of what other players do there is no alternative strategy to be played that will bring more utility to the player.*

A trader can either report its true or a manipulated value to maximizes its payoff.

The quantity of items sold/bought by each seller/buyer is shown as θ_i^S, θ_j^B respectively. The total number of items traded in the auction Θ is calculated as follows:

$$\Theta = \sum_{i=1}^{M} \theta_i^S = \sum_{j=1}^{N} \theta_j^B \tag{1}$$

B. Desired Economic Properties

The four essential principles of a desirable auction mechanism design include optimal allocative efficiency (AE), incentive compatibility (IC), individual rationality (IR) and budget balance (BB) [12].

1) **Optimal Allocative Efficiency (AE):** The outcome allocation of the items shall maximize the social welfare (i.e., the aggregate of all participants' utilities).
2) **Incentive Compatibility (IC):** An auction mechanism is incentive compatible when reporting the true valuation is a dominant strategy for all the traders, i.e., no trader can improve its utility gain from the market by reporting an untruthful value. This is also referred to as by "Truthfulness" and "Strategyproofness" in the literature. IC provides strong incentives for the traders to avoid strategizing the market. The reasons to eliminate strategic behavior from the market are as follows:

a) Strategic behavior of the traders makes the market very complicated to analyze. Especially for a double-auction multi-item market in which there is competition both between the same type of the traders (i.e., seller/seller or buyer/buyer) and opposing type of traders (i.e., seller/buyer) and there is an incentive for them to strategize through untruthful value/quantity reporting to achieve a higher utility.

b) Strategic behavior can impose a substantial social cost on the market as it promotes competitive strategizing. The traders would spend resources to acquire more information about the market and their competitors' preferences, and this consequently will negatively affect their market power, i.e., asks/bids.

3) **Individual Rationality (IR):** All traders have non-negative utility if they participate in the market.
4) **Weak Budget Balance (BB):** The auctioneer does not run at a negative utility. The mechanism is referred to as weakly budget-balanced if the auctioneer does not get a negative utility but it may have a positive utility, and strongly budget-balanced, if the auctioneer's utility is exactly zero. Our desired mechanism is weakly budget-balanced as the auctioneer will get the market surplus as its operation fee.

C. Utility (Payoff)

If the mechanism is IC it means that we have access to the true valuation of the traders, thus it is possible to elicit the amount of utility they gain in the auction. The utility of a seller (u_i^{S}) is the difference between the per-item selling price and the asking price of that seller, times the number of items sold θ_i^{S}:

$$u_i^{\mathrm{S}} = \theta_i^{\mathrm{S}} \times (p^{\mathrm{S}} - v_i^{\mathrm{S}}) \tag{2}$$

The utility of a Buyer (u_j^{B}) is the difference between its submitted bid value and the buying price, times the number of items acquired θ_j^{B}:

$$u_j^{\mathrm{B}} = \theta_j^{\mathrm{B}} \times (v_j^{\mathrm{B}} - p^{\mathrm{B}}) \tag{3}$$

The utility of the auctioneer is the budget surplus which is the difference between the amount paid by the buyers and the amount to be paid to the sellers:

$$u^{Auc} = (p^{\mathrm{B}} \times \sum_{i=1}^{M} \theta_i^S) - (p^{\mathrm{S}} \times \sum_{j=1}^{N} \theta_j^B) \tag{4}$$

and since according to Eq.1

$$\Theta = \sum_{i=1}^{M} \theta_i^S = \sum_{j=1}^{N} \theta_j^B$$

Hence:

$$u^{Auc} = (p^{\mathrm{B}} - p^{\mathrm{S}}) \times \Theta \tag{5}$$

The market model parameters are summarized in Table. I.

TABLE I: Market Model Parameters

Parameter	Descriptions
S_i	i^{th} Seller
B_j	j^{th} Buyer
v_i^{S}	Per-item ask value of i^{th} seller
v_j^{B}	Per-item bid value of j^{th} buyer
q_i^{S}	Quantity of the items offered by i^{th} Seller
q_j^{B}	Quantity of the items demanded by j^{th} buyer
θ_i^{S}	Quantity of the items sold by i^{th} Seller
θ_j^{B}	Quantity of the items bought by j^{th} buyer
Θ_{Pr}	Total No. items traded using the proposed mechanism
Θ_{XU}	Total No. items traded using the Xu et al. mechanism
p^{S}	Sellers' trade price
p^{B}	Buyers' trade price
u_i^{S}	Trade utility of i^{th} Seller
u_j^{B}	Trade utility of j^{th} buyer
u^{Auc}	Trade utility of the Auctioneer

IV. Related Work: Double Auction

Auctions are well-established tools to solve resource allocation problems in telecommunications and computer engineering research. What is common among these research works is their dedication to efficient resource allocation while maintaining the incentives for all the players. In [13] the authors provide an introduction to the auction literature for computer scientists. The applications of the auction in computer science and telecommunication systems range from resource management in cloud networking [14] to digital advertising [15] and wireless spectrum allocation [16]. In [16] the authors have carried out a comprehensive survey of auctions and their application in resource allocation problems in wireless networks. Many different auction mechanisms have been designed to deal with primary (government to operator leasing) and secondary (Inter-operator spectrum trading) radio spectrum market. However, the application of auctions is not limited to spectrum sharing. The authors in [17] have proposed an iterative double auction mechanism for Offloading the traffic of mobile operators to third-party owned Wi-Fi or femtocell access points. Furthermore, auction-based solutions have been proposed to manage the spectrum resources of device-to-device communication in cellular networks [18].

Many different auction mechanisms have been used in the literature and we will not endeavor to survey and present a classification of them and will rely on the classification presented in [16].

Considering the market model of this paper, which is a bilateral trade market with many sellers and many buyers, we chose the double auction model, i.e., two-sided auction that simultaneously matches the sellers and the buyers. Double auctions have received less attention compared to the one-sided auction. The *impossibility theorem* states that no bilateral trading mechanism (e.g., double auction) can simultaneously achieve all of the economic properties described in section III-B and optimal allocative efficiency [19] at the same time. Therefore, an inevitable trade-off is imposed on the system that needs to be addressed by prioritizing the economic properties by how much they would affect the outcome of the system. Thus compromising one of the properties is necessary to achieve the rest, e.g., relaxing the Constraint of allocative efficiency.

Based on the auction classification represented in [16] our market falls into double-sided, sealed-bid, multi-item and single-unit (homogeneous goods) categories (as our mechanism allows each trader to trade multiple items of the same kind).

The most influential work on double auctions was published by McAfee [20]. McAfee acknowledged the impossibility theorem stated by Myerson-Satterthwaite [21] and proposed a dominant strategy double auction that achieves asymptotic efficiency ($\frac{1}{n}$ efficiency loss when n = the number of traders, i.e., it trades the Walrasian equilibrium quantity minus one unit *"see Definition.3 in Section.V"*) while maintaining the desirable economic properties in a single-item single-unit market setting.

Strongly-Budget-Balanced Double-Auction Mechanism [22] sets a single trade price for all traders, in all cases. This may lead to excess supply, which is then handled through a lottery between the sellers. At most one seller, selected at random is excluded from trade. Hence, the expected total-gain-from-trade of SBBA is the same as McAfee's. An advantage of SBBA is that it is strongly budget-balanced. However, this is not an advantage in our market setting since the auctioneer desires a broker fee for conducting the auction.

In [23] a multi-unit double auction (MUDA) mechanism is proposed which splits the market into two sub-markets, by sending each trader to a group with equal probability. Then, in each group, it calculates the Walrasian market price and lets each group's traders to trade with the other group's prices. This way it achieves incentive compatibility and asymptotic efficiency. However, this mechanism is also not a good fit for our market as firstly it is strongly budget balanced as the previous mechanisms, i.e., does not leave any surplus from trades for the auctioneer and secondly, it requires a large number of traders to form two distinct groups or sub-markets, a condition typically not satisfied in multi-tenant PON markets.

In [4] the authors have proposed a market model for secondary market spectrum sharing which has many similarities to our multi-tenant PON market. They successfully applied a McAfee [20] style double auction to their multi-unit auction market. Their mechanism achieves IR, IC, and weak budget balance which is desirable for our market as well. *Xu et al.* also achieved asymptotic efficiency in their mechanism, however, it is not realistic to assume any PON system with such a large number of VNOs to be enough to qualify to be considered an asymptotic setting. Thus, we put our best effort to improve the non-asymptotic efficiency. The preliminary results of this research was presented in OFC 2018 [24] and PIMRC 2017 [25] Conferences where we employed a VickreyClarkeGroves auction (VCG) in the buyers side and uniform price procurement on the sellers side.

In the next section, we propose an auction mechanism that determines the quantity and the price of trade for each trader in a way that firstly requires minimal communication among the trades and the auctioneer and secondly, it achieves higher or at worst the same social welfare for the market compared to a similar mechanism proposed in the literature [4].

V. The Proposed Auction Mechanism

This section describes our sealed-bid homogeneous item double auction mechanism. In a sealed-bid auction all traders simultaneously submit sealed asks/bids to the auctioneer, and no trader is aware of the ask/bid of any other participant. An auction is homogeneous item when the trading items are identical; thus the traders have no preferences over any of the items.

The mechanism provides a matching service to multiple buyers and sellers in a bilateral trade environment. We assume rational traders whose effort is focused on maximizing their payoff by trading the identical items. A market maker, or the auctioneer, is responsible for operating this market and conducting the auctions while having no or incomplete information about the true valuations of the traders. There is a finite number of alternative resource allocation combinations. Each combination would bring a different individual and social payoff for each trader and the whole market. The reason for choosing a sealed-bid auction is that due to the latency constraints in our network and the fact that we need to run the auction in microseconds time scales, we cannot afford multiple rounds of communication among the sellers, auctioneer and the buyers. Therefore, we have to minimize these communications. Using the sealed-bid version of auction helps us with this purpose as it eliminates the need for any additional round of communication among the agents as the traders only send the ask/bid values once along with their BWmap (the suggested allocation schedule).

We assume that there are no restrictions on the sets of buyers and sellers that may trade with one another nor any preferences over trade between any of the traders. The steps of the proposed auction mechanism are as follows:

1) The auctioneer sorts all the sellers/buyers (based on their ask/bid value) so that:

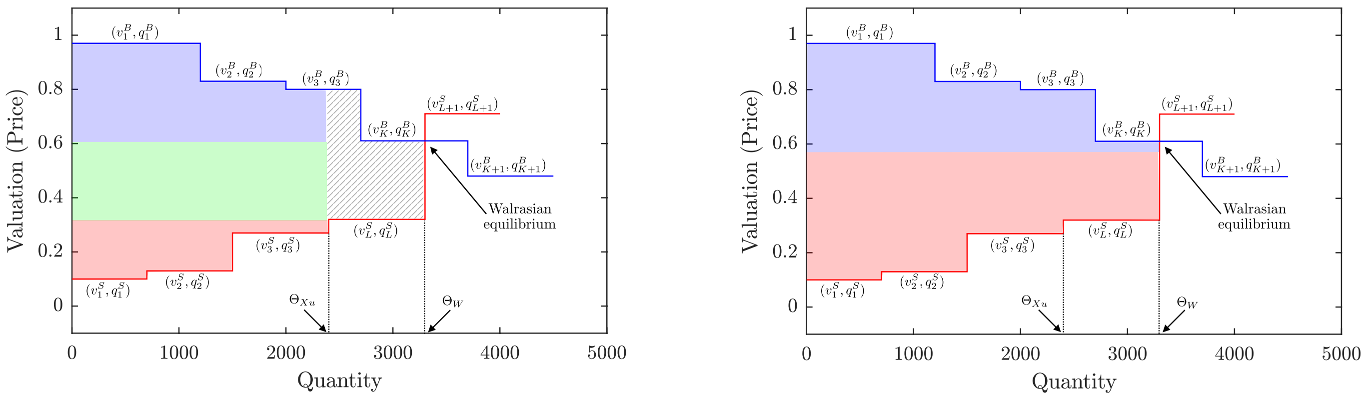
$$\{\tilde{\mathbf{S}} = \{s_1, s_2, ..., s_i\} : v_1^S < v_2^S < ... < v_n^S\} \quad (6)$$

and

$$\{\tilde{\mathbf{B}} = \{b_1, b_2, ..., b_j\} : v_1^B > v_2^B > ... > v_m^B\} \quad (7)$$

Equation 6 shows the sellers arranged in an ascending order and Equation 7 shows the buyers sorted in a descending order by their per-item valuation (v).

2) The auctioneer discovers the Walrasian equilibrium quantity as shown in Fig. 3 as Θ_{w}. Fig. 3 depicts the discrete supply and demand graph for one instance of the auction (one round of the auction for one frame of the PON) representing the trades' valuation and quantity. In Fig. 3 each step in the red line represents a seller and each step in the blue line is a buyer. In this example there are five sellers and five buyers in the market and $\gamma \in [v_L, v_K]$. As shown in Fig. 3a, the gray area, representing the trading utility, is sacrificed to achieve truthfulness. In contrast, our proposed trade reduction technique depicted in Fig. 3 saves this amount of wasted



(a) Xu et al. Double-Auction

(b) Proposed Double-Auction

Fig. 3: Discrete supply-demand graph of the double auction

utility while maintaining the truthfulness. In Fig. 3 the blue, green and red area represent buyers', auctioneer's and the sellers' utility from the auction. As it can be seen in Fig. 3, for this particular instance of the market our proposed mechanism brings zero utility for the auctioneer. In general however the auctioneer's overall utility will be the surplus between buyers' payment and the sellers' receivables goes to the auctioneer (zero when $\gamma \in [v_L, v_K]$ and $(p^{\mathrm{B}} - p^{\mathrm{S}}) \times \Theta$ when $\gamma \notin [v_L, v_K]$).

Definition 3. *(Walrasian (Competitive) Equilibrium). In the context of our double auction market, the "Walrasian Equilibrium" is the point in the supply-demand plot in which the supply equals the demand. This point specifies the maximum quantity of feasible trades in which the sellers' price is less than the buyers' price. In other words, this is the upper-bound utilization of the market.*

The "Walrasian Equilibrium" defines another important factor, the "Walrasian Price" which, if the trade is conducted brings positive payoff for both the supplier and the demander and also balances the budget. i.e. finds the biggest (L, K) in which:

$$v_K^B \geq v_L^S \text{ and } v_{K+1}^B \leq v_{L+1}^S, \tag{8}$$

and

$$\sum_{j=1}^{\mathrm{K}} q_j^B \leq \sum_{i=1}^{\mathrm{L}} q_i^S. \tag{9}$$

Thus the last trading seller and buyer in the Walrasian equilibrium are called S_L, B_K respectively. The Walrasian equilibrium realizes the *Pareto efficiency*. A resource allocation decision is referred to as Pareto efficient if it is impossible to reallocate the resources in a way that makes one of the agents better off without making others worse. This quality makes the Walrasian allocation a suitable benchmark of efficiency in economic analysis.

3) In order to achieve dominant strategy truthful value reporting we have to decouple the trade price of the sellers and buyers from their reported value. This is achievable through "Trade Reduction".

Definition 4. *Trade Reduction: A technique in which the least efficient trade in the market is sacrificed so the other traders can trade on their reported value, thus their reported valuation does not affect their payments, i.e., they have no incentive to report untruthful values (IC).*

Thus q_r trades will be removed from the market. In Fig. 3 the reduced area is shown in gray. Obviously, it is possible that by removing q_r trades from the market we may have to eliminate more than two traders (completely or partially) from the market.

In our proposed mechanism, the total number of items sold by the sellers, which is equal to the total number of items bought, by the buyers is represented as Θ_{Pr}. The value of Θ_{Pr} directly affects the utilization of the PON link.

$$\Theta_{Pr} = \min\left(\sum_{i=1}^{L} q_i, \sum_{j=1}^{K} q_j\right) \tag{10}$$

The quantity of reduced trades is defined as q_R:

$$q_R = \Theta_W - \Theta_{Pr} \tag{11}$$

The amount of efficiency (utility) sacrificed in market due to this trade reduction is:

$$\textit{Efficiency loss} = q_R \times (v_K - v_L) \tag{12}$$

Our contribution in this paper is to try and minimize q_R without loosing the economic properties. We use the technique used in [20] which uses the values of S_{L+1} (the strongest non-trading seller) and B_{K+1} (the strongest non-trading buyer) to determine the traders payment only and only if:

$$\gamma = \frac{1}{2} \times (v_{L+1}^S + v_{L+1}^B) \in [v_i^S, v_j^B] \tag{13}$$

thus since v_L^S, v_K^B, and in general none of the trading players, do not play a role in the price determination, i.e., there is no need to eliminate any of the traders including S_L and B_K from the market, therefore $q_R = 0$. In this case the sellers and the buyers both trade in $\gamma = p^S = p^B$.

Definition 5. *Weakest/Strongest Trader: In the context of market power, the weakest traders are the seller with the highest asking price v_i^S and the buyer with the lowest* *bid v_j^B. On the other side, strongest traders are the seller with the lowest ask and the buyer with the highest bid.*

In Fig.3 the blue area is the sellers utility, the green area is the auctioneers and the red is the buyers. The gray area is the amount of lost efficiency, e.g., when the trade reduction is applied.

Algorithm 1: Multi-Item Double Auction

Sort sellers ascending so $v_1^B > v_2^B > ... > v_m^B$
Sort buyers descending so $v_1^S < v_2^S < ... < v_n^S$
Find $max(S_L, B_K)\ \forall\ v_L < v_K$ **and** $\sum_1^{\mathrm{K}} q_j^B \leq \sum_1^{\mathrm{L}} q_i^S$
$\gamma = \frac{1}{2} \times (v_{\mathrm{L+1}} + v_{\mathrm{K+1}})$
if $\gamma \in [v_L, v_K]$ **then**
    $\Theta_{Pr} = \min(\sum_1^{i=L} q_i, \sum_1^{j=K} q_j)$
    $p^B = p^S = \gamma$
else if $\gamma \notin [v_L, v_K]$ **then**
    $\Theta_{Pr} = \min(\sum_1^{i=L-1} q_i, \sum_1^{j=K-1} q_j)$
    $p^B = v_K$
    $p^S = v_L$

TABLE. IIa presents the numerical information (the traders' prefrences) used in Fig. 3 and the different market results using the Xu et al. (TABLE. IIb) proposed mechanism (TABLE. IIc).

VI. Theoretical Proofs

1) Truthfulness: We have to proof that no trader can achieve higher utility by reporting a manipulated value which can be determined by market monitoring techniques and prediction tools such as machine learning etc.

Lemma 1. *No buyer can win more items by reporting an untruthfully lower value i.e.* $\{\forall\ v_j' < v_j | \theta_j' \leq \theta_j\}$.

Proof: The value of the θ_j' depends on whether if the v_j' changes the position of B_j in the sorted buyers' list.

Case 1.a If B_j reports a $v_j' < v_j$ and by doing so its position in the sorted buyers list does not change then by design the outcome of the auction remains the same so the quantity will not change i.e. ($\theta_j' = \theta_j$).

TABLE II: An instance of the double auction market.

Sellers	700@0.10	800@0.13	900@0.27	900@0.32	700@0.71
Buyers	1200@0.97	800@0.83	700@0.80	1000@0.61	800@0.48

(a) Sellers and buyers and their quantity to sell/buy@ask/bid price

Sellers	700@0.32	800@0.32	900@0.32	0	0
Buyers	1200@0.61	800@0.61	400@0.61	0	0
					$\Theta = 2400$

(b) Sellers and buyers and $\theta_i^S/\theta_j^B @ p^S/p^B$ Xu et al. mechanism

Sellers	700@0.595	800@0.595	900@0.595	900@0.595	0
Buyers	1200@0.595	800@0.595	700@0.595	600@0.595	0
					$\Theta = 3300$

(c) Sellers and buyers and $\theta_i^S/\theta_j^B @ p^S/p^B$ proposed mechanism

Case 1.b If the reported v_j' changes the position of the buyer in the sorted buyers list, since its position will be downgraded the θ_j' will be at best case (e.g. if $j \leq k$) equal to θ_j and in the worst case (e.g. if $j = k + 1$) zero i.e. $\theta_j' \leq \theta_j$.

Thus we have proven that if B_j reports an untruthful bid $v_j' < v_j$ it cannot increase its trade quantity, i.e. $\nexists\ {v'}_j^B < v_j^B \rightarrow (\theta_j' \leq \theta_j)$. ∎

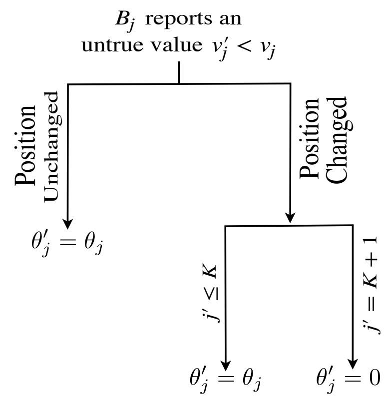


Fig. 4: The proof tree for Lemma 1

Lemma 2. *No buyer can decrease its per-item payment value by reporting an untruthfully lower value i.e.* $\{\forall\ v_j' < v_j | p_j' \geq p\}$.

Proof: Lets consider that B_j reports an untruthful bid $v_j' < v_j$.

Case 2.a If the new v_j' does not change the position of B_j in the sorted buyers' list then the auction outcome remains unchanged and will not change, i.e. $p' = p$.

Case 2.b If the new v_j' changes the position of B_j in the sorted buyers' list, the following cases can occur:

Case 2.b.1 If the new $j' < K$ then the p remains unchanged , i.e. $p' = p$.

Case 2.b.2 If $j' = K$ and $\gamma \in [v_L, v_K]$ then again p remains unchanged as it is equal to γ which is independent of the v_j', i.e. $p' = p$.

Case 2.b.3 If $j' = K$ and $\gamma \notin [v_L, v_K]$ then B_j' does not win any items, i.e. $p_j' = 0$.

Case 2.b.4 Finally if $j' > K$ then B_j' does not win any items, i.e. $p_j' = 0$.

Thus we have proven that if B_j reports an untruthful bid $v_j' < v_j$ it cannot lower its trade price, i.e. $\nexists\ {v'}_j^B < v_j^B \rightarrow (p' \leq p)$. ∎

Theorem 1. *Bid Independence: There is no ${v'}_j^B < v_j^B$ which B_j can report and by doing so gain more utility ($u_j' \leq u_j$) given that all the other bids and asks remain unchanged, i.e.* $\nexists\ {v'}_j^B < v_j^B \rightarrow (u_j' \leq u_j)$.

Proof: According to equation 3 the utility of a buyer only increases if the quantity of trade increases or the price to pay decreases or both. By proving lemma 1 and lemma 2 we have proven that there is no way for any buyer to manipulate the market by reporting a lower value than its true valuation and gain higher utility by doing so i.e. $\{\forall\ v_j' < v_j | u_j' \geq u_j\}$. Thus

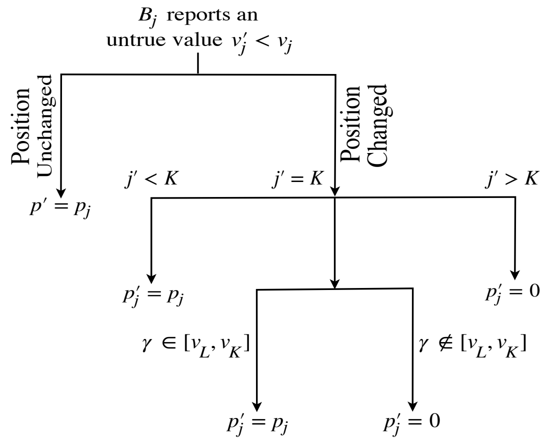


Fig. 5: The proof tree for Lemma 2

the final utility of a buyer is independent from the bid that it submits if not higher. ■

Corollary 1. *According to theorem 1, the final utility is independent of the submitted bid. Thus it is a weakly dominant strategy for every buyer to report their true value and have a higher chance at winning since at worst the utility of truthful reporting is equal to that of a shaded (manipulated) bid.*

Corollary 2. *The dominant strategy truthfulness for the sellers can be proven in the same way as of Corollary 1.*

It is noteworthy that if a seller/buyer reports a lower/higher value they will move down in the sorted traders list, thus lowering the chance of trading more items.

A. Individual Rationality (IC)

Theorem 2. *The proposed mechanism satisfies all traders' individual rationality.*

Proof: The proposed mechanism is IC since by design it will not sell (buy) any item unless the trade price is higher (lower) than the seller's (buyer's) reported value. We consider that the VNOs are rational entities and would not report smaller values than their true value if they are sellers or bigger values than their true value if they are buyers. ■

B. Weak Budget Balance (BB)

Theorem 3. *The proposed mechanism is BB.*

Proof: The suggested theorem is invalid when it is possible for the auctioneer to run a budget deficit. We will show that our mechanism does not allow that to happen. According to formula 4 to get a negative u_{Auc} the following relationship must hold:

$$(p^{\text{B}} \times \sum_{1}^{n} \theta_j^{\text{B}}) < (p^{\text{S}} \times \sum_{1}^{m} \theta_i^{\text{S}})$$

that equivalently leads to the following inequality:

$$\frac{p^{\text{B}}}{p^{\text{S}}} < \frac{\sum_1^n \theta_j^{\text{B}}}{\sum_1^m \theta_i^{\text{S}}}$$

We know from the mechanism that $\sum_1^n \theta_j^{\text{B}} = \sum_1^m \theta_i^{\text{S}} = \Theta$, i.e. the total number of sold items is equal to the total number of items bought. Thus:

$$\frac{p^{\text{B}}}{p^{\text{S}}} < 1$$

according to our proposed mechanism $p^{\text{B}} > p^{\text{S}}$, i.e. the buyers' trade price is always higher than the sellers' trade price. Consequently, the above inequality does not hold since it is in contradiction with the fact that $p^{\text{B}} > p^{\text{S}}$. Thus, we have proved that the u_{Auc} can never be negative. ■

VII. Experimental Results

This section is dedicated to evaluating and analyzing the allocative efficiency of the proposed mechanism and comparing it to prior work. We measure the allocative efficiency by two factors:

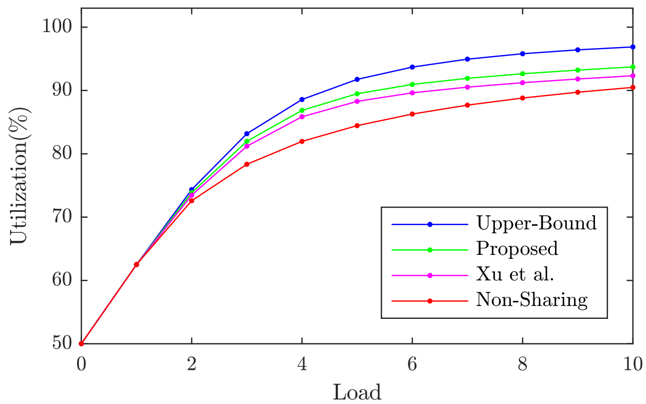
1) The total number of items traded in one round of auction (Equation 1). This factor directly determines the proportion of the PON bandwidth that is being shared among the VNOs. Thus it can clearly reflect the effect of auctioning the capacity on PON's efficiency.
   To compare the results of different mechanisms we use the Walrasian equilibrium trade quantity as a baseline (upper-bound) since this is the maximum number of rational trades in the market. The closest the results are to the upper-bound the better. A rational trade is a trade in which the buyer's bid is strictly higher than the sellers, and the supply quantity is larger or smaller than the demand quantity.
2) The social welfare, which is a factor representing the aggregate benefit brought to all the parties involved in the market. The social welfare is calculated by summing the utilities of the trading traders and the auctioneer in every round of auction. The social welfare can clarify whether the mechanism has been successful in redistributing the bandwidth from the sellers who value the items the least to the buyers with the highest valuation and therefore maximizing the total profit generated by the market. The Social Welfare is calculated as follows:

$$SW = \sum_{i=1}^{i=L} u_i^S + \sum_{j=1}^{j=K} u_j^B + u_{Auc} \tag{14}$$

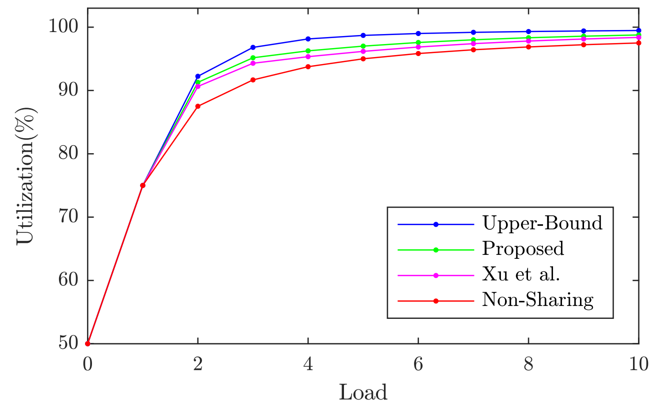
   In which the SW is the social welfare of the system in each round of auction.

A. Simulation Setup

We consider an XGS-PON [5] with 10 Gbit/s (nominal line rate of 9.95328 Gbit/s) symmetrical capacity. We simulate a market with ten VNOs each with an equal share of the upstream bandwidth, i.e., 995.328 Mbit/s≈1-Gbit/s. This translates to 972 blocks (one block is 16 bytes as defined in the standard [5] as the finest granularity for upstream allocation) or blocks per frame (125 μs) per VNO. Each VNO will ask for a number of blocks depending on its users' instantaneous demand. This number determines that if the VNO is a seller

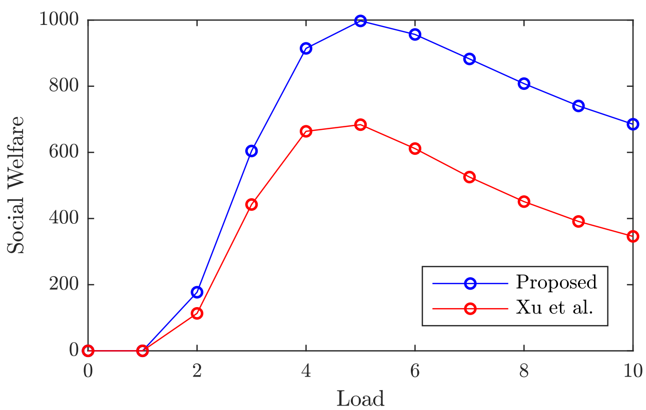


(a) Unbalanced Load

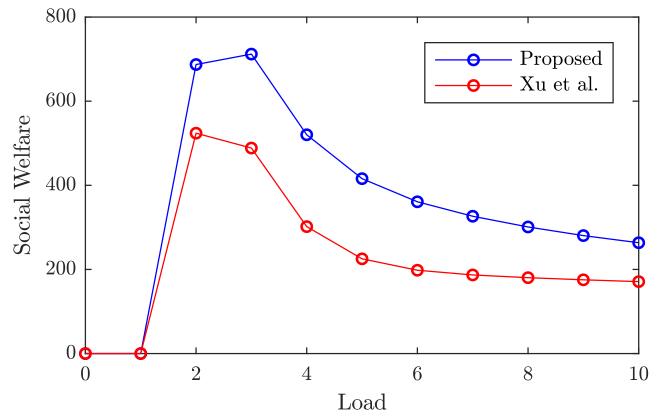


(b) Balanced Load

Fig. 6: Simulation results of our DBA auctions, showing how the auction mechanism increases PON utilization.



(a) Unbalanced Load



(b) Balanced Load

Fig. 7: Simulation results of our DBA auctions, showing how the auction mechanism increases the average Social Welfare.

(if asking for a lower than the pre-defined share), a buyer (if asking for higher) or a non-trader (if asking for the exact same amount).

B. Simulation Results

In this section, we report the market simulation results.

Figure 6 illustrates the performance comparison of the proposed mechanism with non-sharing and *Xu et al.* [4] mechanism. The *"Upper-Bound"* is the maximum reachable utilization while overlooking the economic properties. As we mentioned in the previous sections "*Xu et al.*" is a mechanism that works similar to our proposed mechanism with a difference that it always removes S_L, B_K. The *"Non-sharing"* is when the VNOs do not share their excess bandwidth (i.e., no auction happens and all the excess bandwidth is wasted).

Figures 6a, and 6b depict the utilization (averaged over the simulation time) of each mechanism in the unbalanced (randomly weighted load) and balanced (equally weighted load) network loads respectively. The *"Upper-Bound"* achieves the highest utilization as it ignores the truthfulness and puts a naive trust on the VNOs to report their values truthfully, thus does not remove any trades from the market. However, this is not acceptable since in such conditions the traders do not have any incentive to report true values and will potentially try to manipulate the market by reporting untrue values. This may lead to a situation where no trader gets to trade since they all are greedily trying to maximize their own utility without considering the others' welfare, e.g., ask prices are too high and bids are too low, so no trade happens. The horizontal axis represents the average incoming load of each VNO, and the vertical axis is the utilization of each mechanism in a given load. The numerical results of the simulation are given in Table III and Table IV. According to our results, in both balanced and unbalanced load, the proposed mechanism outperforms *Xu et al.* mechanism as its trade reduction technique allows more trades to happen.

The main improvement of our algorithm is however in the social welfare (aggregate market utility averaged over the simulation time) generated by auctioning the excess capacity of the PON, show in Fig. 7. The Unit for utility is one block (XGS-PON frame block), e.g., the utility of 1000 in the Fig. 7a means that in average over 10 seconds of simulation time $\approx$ 1000 additional frame blocks worth of utility is gained when using the proposed mechanism. The results in Fig. 7 provides further support for the hypothesis that the proposed mechanism achieves higher social welfare compared to that of the Xu et al. [4]. The explanation for this difference lays in our trade reduction technique that reduces fewer trades compared to the Xu et al. [4]. We recognize that the significant improvements in Fig. 7a that reaches up to $\approx 40\%$ may seem unrealistic at first glance. To explain this, it is important to note that as

TABLE III: Unbalanced Load Utilization

Load	Non-Sharing	*Xu et al.*	Proposed	Upper-Bound
1	75.01	75.01	75.01	75.01
2	87.51	90.64	91.28	92.24
4	93.75	95.34	96.26	98.14
6	95.84	96.86	97.58	99.00
8	96.88	97.81	98.33	99.31
10	97.51	98.38	98.77	99.47

TABLE IV: Balanced Load Utilization

Load	Non-Sharing	*Xu et al.*	Proposed	Upper-Bound
1	62.51	62.51	62.51	62.51
2	72.59	73.46	73.86	74.35
4	81.96	85.85	86.86	88.58
6	86.28	89.63	90.96	93.70
8	88.81	91.23	92.65	95.80
10	90.49	92.33	93.72	96.88

the network load increases, the number of eligible traders is reduced since it becomes less likely for a VNO to have any excess resources to share. Therefore, it is likely that the trade eliminated by the trade reduction algorithm might be the only efficient trade, i.e., the trade reduction technique might ban the only feasible trade thus leaving the market with no additional social welfare.

VIII. Conclusion

In this work we have addressed the sharing incentive issue of the network operators who coexist and co-operate on a shared PON. We have provided the lacking incentive mechanism through monetization of the excess resources and then modeling the multi-tenant PON as a bilateral trade market. Furthermore, we have proposed a new sealed-bid, multi-item, double auction mechanism to efficiently allocate the resources while maximizing the social welfare of the market. We have proven that our proposed algorithm is compatible with the VNOs' incentives and guarantees positive a budget for the InP. The results from the market simulation show that the proposed mechanism outperforms the double auction mechanism proposed by Xu et al. [4], as our trade reduction mechanism scarifies fewer trades to achieve the crucial economic properties. Finally, these achievements are reached while the mechanism adds no additional communication overhead to the system due to its single rounded nature. Multi-tenancy could potentially facilitate new partnership and co-investment models for network operators. In this papers, we addressed one such model in which a trusted infrastructure provider is the sole provider of the resources. However, more complex sharing models are yet to be addressed as new network ownership/operation models are introduced thanks to the network virtualization.

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